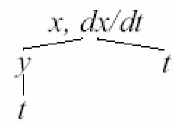
MATH 223.3 2006-2007 (01) (03) (05)

QUIZ #4 - Solutions
Each question is worth 5 points - total = 15.
Integer scores *only*.

#1

From $\dfrac{dx}{dt} = \dfrac{\partial x}{\partial y}\dfrac{dy}{dt} + \dfrac{\partial x}{\partial t} = (2y+t)(t^2e^t + 2te^t) + (y-2t)$

$$= te^t(t^2 + 2yt + 4y + 2t) + y - 2t,$$



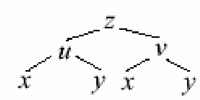
$$\frac{d^2x}{dt^2} = \frac{\partial}{\partial y}\left(\frac{dx}{dt}\right)\frac{dy}{dt} + \frac{\partial}{\partial t}\left(\frac{dx}{dt}\right)$$

$$= [te^t(2t+4)+1](t^2e^t + 2te^t) + [te^t(2t+2y+2) + (te^t + e^t)(t^2 + 2yt + 4y + 2t) - 2]$$

$$= 2t^2(t+2)^2e^{2t} + (6t^2 + 6t + 8ty + t^3 + 2yt^2 + 4y)e^t - 2.$$

#2

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y} = [3u^2v + v\cos(uv)]\frac{\partial u}{\partial y} + [u^3 + u\cos(uv)]\frac{\partial v}{\partial y}$$



If we set $F(x,u,v) = e^u\cos v - x$, $G(y,u,v) = e^u \sin v - y$, then

$$\frac{\partial u}{\partial y} = -\frac{\dfrac{\partial(F,G)}{\partial(y,v)}}{\dfrac{\partial(F,G)}{\partial(u,v)}} = -\frac{\begin{vmatrix} F_y & F_v \\ G_y & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} = -\frac{\begin{vmatrix} 0 & -e^u\sin v \\ -1 & e^u\cos v \end{vmatrix}}{\begin{vmatrix} e^u\cos v & -e^u\sin v \\ e^u \sin v & e^u\cos v \end{vmatrix}} = \frac{e^u\sin v}{e^{2u}} = e^{-u}\sin v,$$

and $$\frac{\partial v}{\partial y} = -\frac{\dfrac{\partial(F,G)}{\partial(u,y)}}{\dfrac{\partial(F,G)}{\partial(u,v)}} = -\frac{\begin{vmatrix} F_u & F_y \\ G_u & G_y \end{vmatrix}}{e^{2u}} = -\frac{\begin{vmatrix} e^u\cos v & 0 \\ e^u\sin v & -1 \end{vmatrix}}{e^{2u}} = e^{-u}\cos v.$$

Thus, $$\frac{\partial z}{\partial y} = [3u^2v + v\cos(uv)]e^{-u}\sin v + [u^3 + u\cos(uv)]e^{-u}\cos v.$$

#3

Since parametric equations for the curve are $x = t$, $y = -\sqrt{t^2-3}$, $z = t$, a tangent vector at $(2,-1,2)$ is $\mathbf{T}(2) = \left(1, \dfrac{-t}{\sqrt{t^2-3}}, 1\right)_{|t=2} = (1,-2,1)$. At the point $(2,-1,2)$ then,

$$D_{\mathbf{T}}f = \nabla f_{|(2,-1,2)} \cdot \hat{\mathbf{T}} = (2xy + y^3z, x^2 + 3xy^2z, xy^3)_{|(2,-1,2)} \cdot \frac{(1,-2,1)}{\sqrt{1+4+1}}$$

$$= (-6, 16, -2)\cdot\frac{(1,-2,1)}{\sqrt{6}} = \frac{-40}{\sqrt{6}}.$$

As usual, $\mathbf{v}^{\wedge}$ is a unit vector in the direction of $\mathbf{v}$.